

Hong Kong Mathematics Olympiad (2004 – 2005)

Heat Event (Individual)

香港数学竞赛 (2004 – 2005)

初赛项目(个人)

除非特别声明，答案须用数字表达，并化至最简。

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

1. 若  $p$  和  $q$  是正整数且  $\frac{96}{35} > \frac{p}{q} > \frac{97}{36}$ ，求  $q$  最小可能的值。

Suppose  $p, q$  are positive integers and  $\frac{96}{35} > \frac{p}{q} > \frac{97}{36}$ , find the smallest possible value of  $q$ .

2. 已知  $x = 2005$  及  $y = |4x^2 - 5x + 9| - 4|x^2 + 2x + 2| + 3x + 7$ ，求  $y$  的值。

Given that  $x = 2005$  and  $y = |4x^2 - 5x + 9| - 4|x^2 + 2x + 2| + 3x + 7$ , find the value of  $y$ .

3. 若  $x$  是实数且满足  $\left(\sqrt{5+2\sqrt{6}}\right)^x + \left(\sqrt{5-2\sqrt{6}}\right)^x = 10$ ，求  $x$  的最小可能的值。

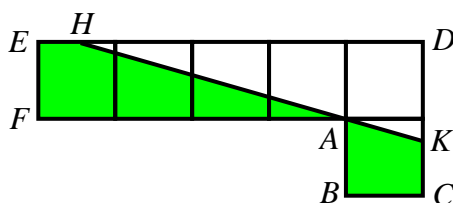
If  $x$  is a real number satisfying the equation  $\left(\sqrt{5+2\sqrt{6}}\right)^x + \left(\sqrt{5-2\sqrt{6}}\right)^x = 10$ , find the smallest possible value of  $x$ .

4. 设  $t$  为实数且满足  $(1 + \sin t)(1 + \cos t) = \frac{5}{4}$ 。若  $N = \sin t + \cos t$ ，求  $N$  的值。

Let  $t$  be a real number satisfying  $(1 + \sin t)(1 + \cos t) = \frac{5}{4}$ . If  $N = \sin t + \cos t$ , find the value of  $N$ .

5. 如图一， $ABCDEF$  是由六个正方形所组成的“L形”图案。 $HAK$  是一直线，阴影部分的面积是  $ABCDEF$  的面积 的  $\frac{1}{2}$ 。若各小正方形的边长是 1 cm， $HK$  的长度是  $m$  cm，求  $m$  的值。

In Figure 1,  $ABCDEF$  is a “L shape” figure formed by six squares.  $HAK$  is a straight line and the area of the shaded region is equal to  $\frac{1}{2}$  of the area of  $ABCDEF$ . If the length of each small square is 1 cm and the length of  $HK$  is  $m$  cm, find the value of  $m$ .



图一

Figure 1

6. 设  $n$  是自然数，直线  $nx + (n+1)y = \sqrt{2}$  与两坐标轴所围成的三角形的面积是  $S_n$ 。若  $K = S_1 + S_2 + \cdots + S_{2005}$ ，求  $K$  的值。

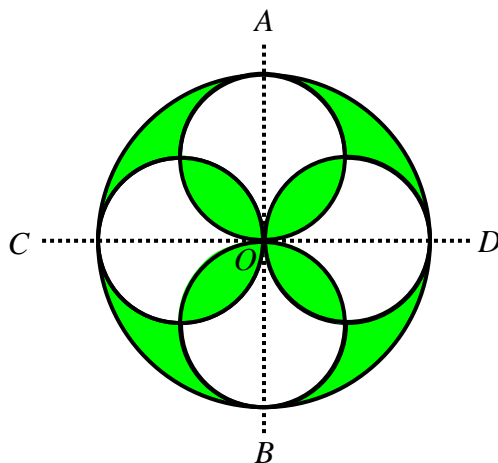
Let  $n$  be a natural number, the area of the triangle bounded by the line  $nx + (n+1)y = \sqrt{2}$  and the two coordinate axes is  $S_n$ . If  $K = S_1 + S_2 + \cdots + S_{2005}$ , find the value of  $K$ .

7. 设  $[x]$  表示不大于  $x$  的最大整数，例如  $[2.5] = 2$ 。若  $M = \sum_{n=1}^{1024} [\log_2 n]$ ，求  $M$  的值。

Let  $[x]$  be the largest integer not greater than  $x$ , for example,  $[2.5] = 2$ . If  $M = \sum_{n=1}^{1024} [\log_2 n]$ , find the value of  $M$ .

8. 如图二， $AB$  垂直于  $CD$ ，其交点  $O$  是大圆的圆心，而四个小圆的圆心分别在  $AB$  和  $CD$  上。已知大圆的半径是  $1\text{ cm}$ ，四个小圆的半径是  $\frac{1}{2}\text{ cm}$ 。若阴影部分的面积是  $R\text{ cm}^2$ ，求  $R$  的值。  
(取  $\pi = 3$ )

In Figure 2,  $AB$  is perpendicular to  $CD$ , their intersection point  $O$  is the centre of the large circle and the centers of the four circles lies on either  $AB$  or  $CD$ . Given also that the radius of the large circle is  $1\text{ cm}$  and the radius of each of the four small circles is  $\frac{1}{2}\text{ cm}$ . If the area of the shaded region is  $R\text{ cm}^2$ , find the value of  $R$ . (take  $\pi = 3$ )



图二

Figure 2

9. 已知  $60^a = 3$  及  $60^b = 5$ 。若  $R = 12^{\frac{1-a-b}{2(1-b)}}$ ，求  $R$  的值。

Given that  $60^a = 3$  and  $60^b = 5$ . If  $R = 12^{\frac{1-a-b}{2(1-b)}}$ , find the value of  $R$ .

10. 已知 2005 年 1 月 29 日是星期六，那么 2008 年 1 月 29 日是星期几？

Given that 29<sup>th</sup> January 2005 is Saturday, on what day is 29<sup>th</sup> January 2008 ?